

Computational Statistics and Data Analysis (MVComp2)

Exercise 2

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Due Nov. 2, 2023, 23:59

1 Coin-tossing game (2 points)

You play a game that consists of tossing two coins. You win €1 if both coins land on tails, you win €2 if both coins land on heads, and lose €1 otherwise.

- (a) Calculate the mean and variance of your winnings on a single play of the game.
- (b) What is the fair price to play this game (i.e., payoff and cost of playing have mean 0)?

2 Expectations and variances (3 points)

Let X, Y be discrete random variable and a, b be constants. Prove the following relations:

- (a) $\text{Var}[aX + b] = a^2\text{Var}[X]$
- (b) $E[X] = E_Y[E_X[X|Y]]$
- (c) $\text{Var}[X] = E_Y[\text{Var}[X|Y]] + \text{Var}[E_X[X|Y]]$

3 Covariance and correlation (2 points)

Prove that the correlation coefficient, ρ , is bounded by -1 and 1.

4 Correlation Between CO₂ levels and Earth's surface temperature (3 points)

You set out to investigate the correlations between mean CO₂ levels and Earth's surface temperature over the last few decades. Two datasets are available:

1. Mean monthly CO₂ levels from the Mauna Loa Observatory dataset, which provides a continuous record from 1958 to the present. CSV file `monthly_in_situ_co2_mlo.csv` available at:

https://scrippsco2.ucsd.edu/data/atmospheric_co2/primary_mlo_co2_record.html

2. Global mean surface temperature datasets, available from NASA's Goddard Institute for Space Studies. CSV file of "Global-mean monthly, seasonal, and annual means" available at:

<https://data.giss.nasa.gov/gistemp/>

Procedure:

- Collect the data for the same time frame.
 - Clean the data of any outliers or missing values.
 - Calculate annual means for both datasets.
- (a) Determine the (Pearson) correlation coefficient between annual CO₂ levels and temperature deviation.
 - (b) Visualize the correlation using a parity plot (i.e., temperature deviation vs. CO₂ levels.)

Hint: If using Python, you may find the following `pandas` functions useful: `read_csv`, `groupby`, `merge_asof`.